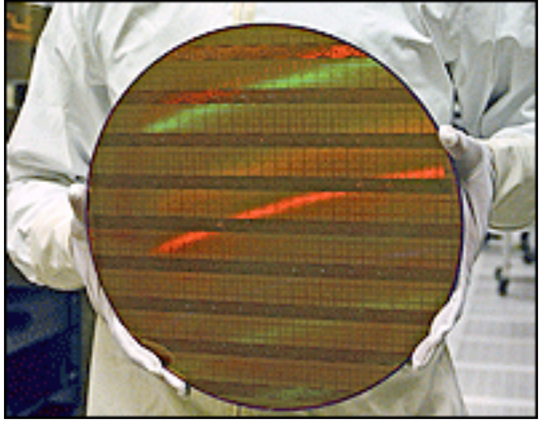
Mesoscopic Electronics

ChiiDong Chen, Institute of Physics, Academia Sinica
中央研究院 物理研究所 陳啟東

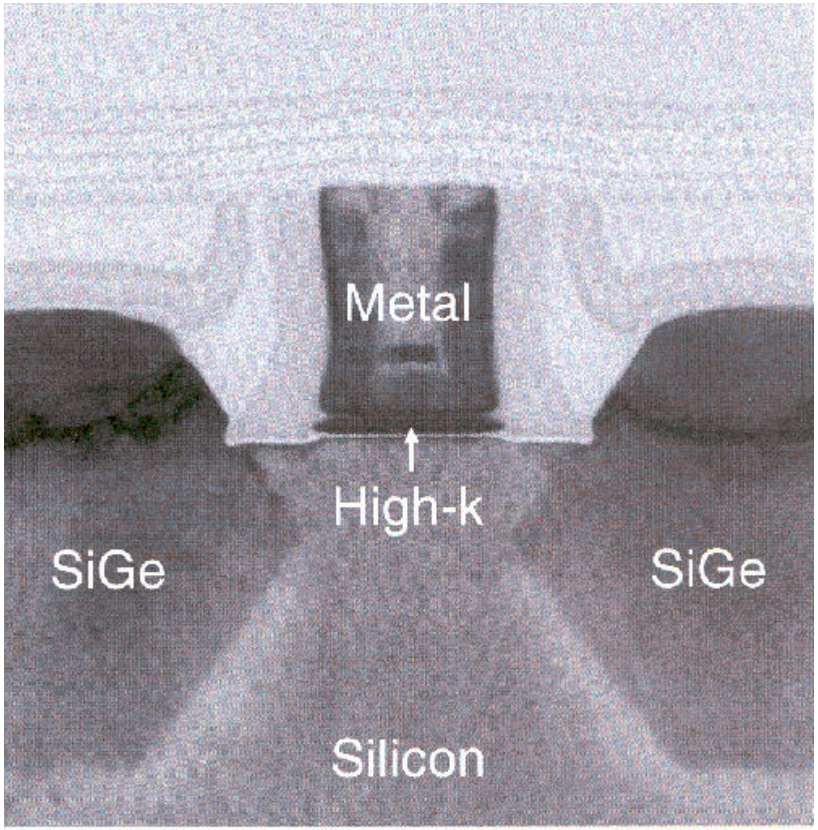
chiidong@phys.sinica.edu.tw.
http://www.phys.sinica.edu.tw/~quela/

Cheng-Kung University, Tainan, Taiwan
Tokyo Institute of Technology, Tokyo, Japan
Chalmers University, Gothenburg, Sweden

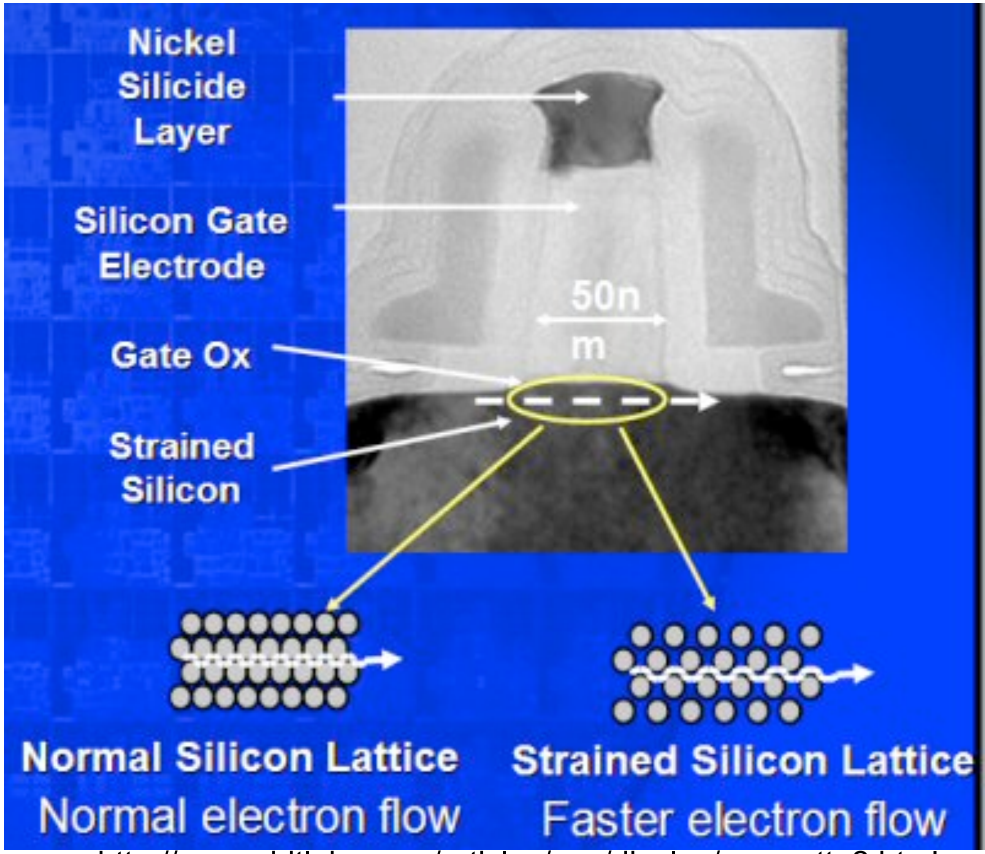
Present-day semiconductor industry



Scientific American, January 30, 2007

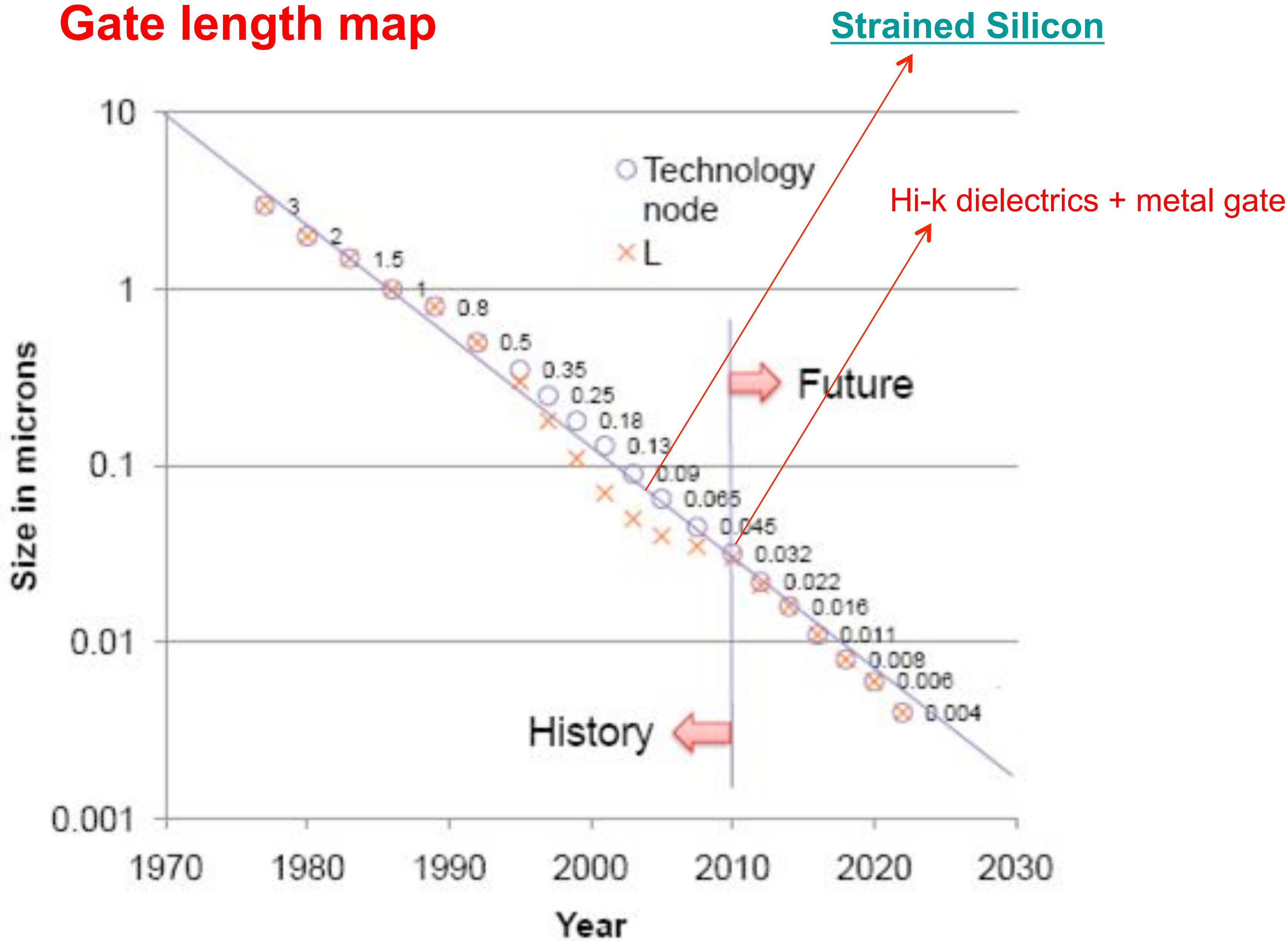


http://channel.hexus.net/content/item.php?item=12389



http://www.xbitlabs.com/articles/cpu/display/prescott_2.html

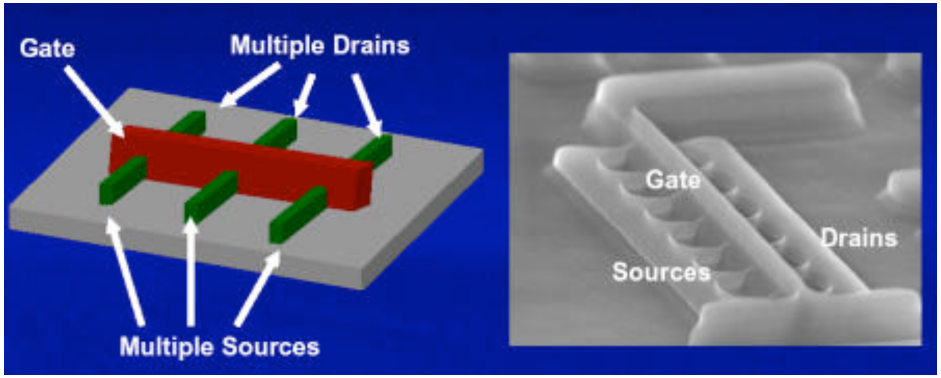
Gate length map



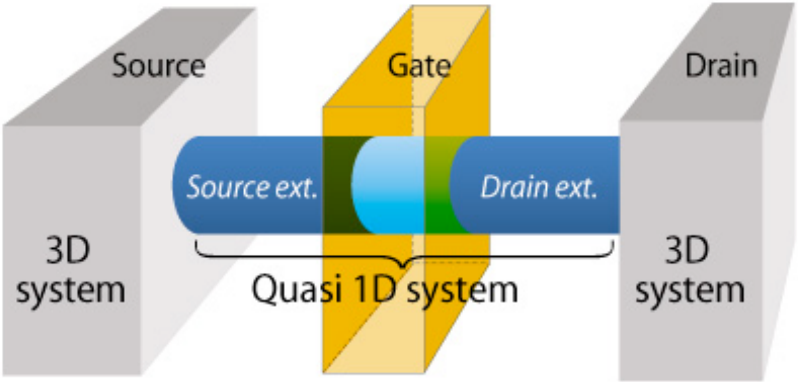
http://theasicguy.com/2010/06/ Michael Keating, SNUG 2010

Latest development and future technologies

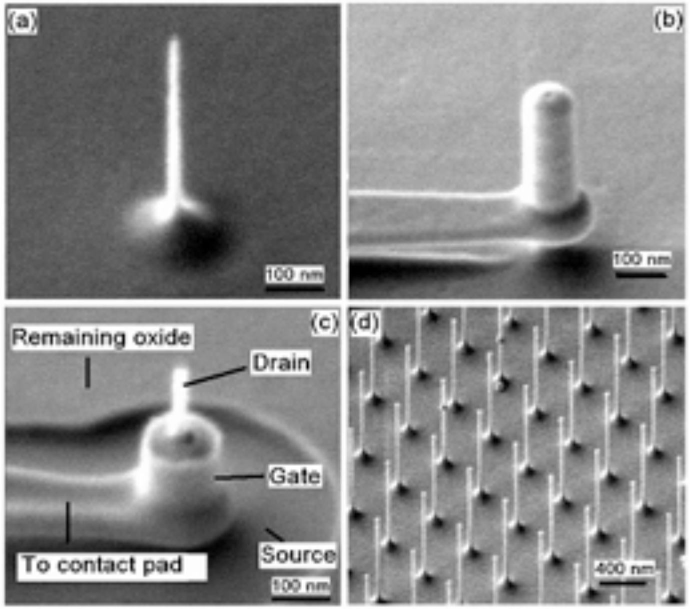
FinFET or Multi-gate FET (MuGFET)



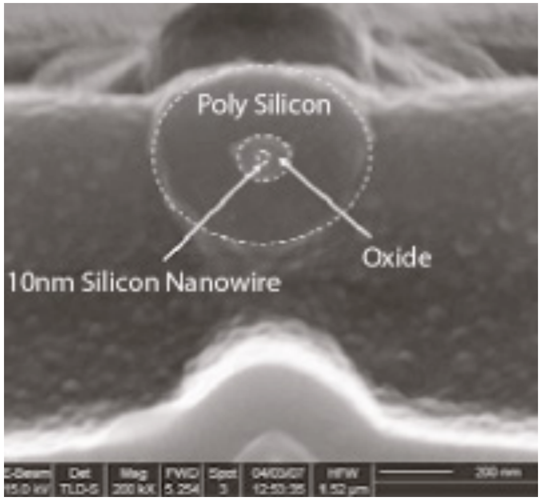
Gate-All-Around Field-Effect-Transistors



Vertical Silicon-Nanowire Formation

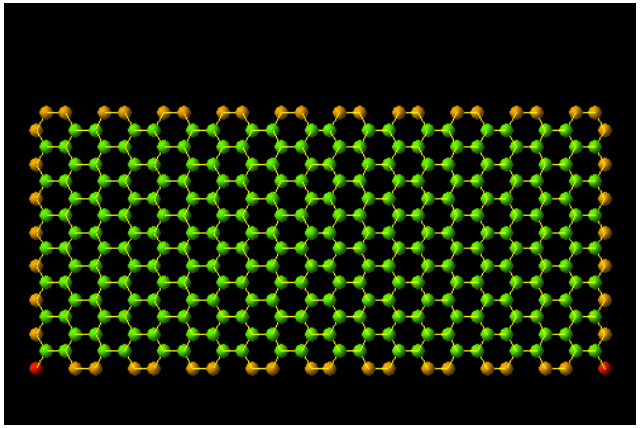


http://www.ime.a-star.edu.sg/html/newsrelease/2008/Shortcuts-May08Jun08.htm

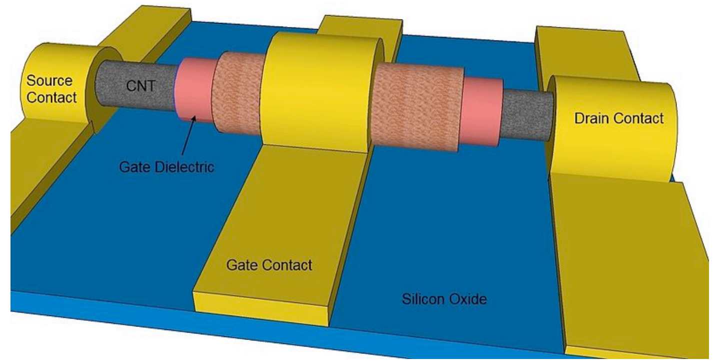


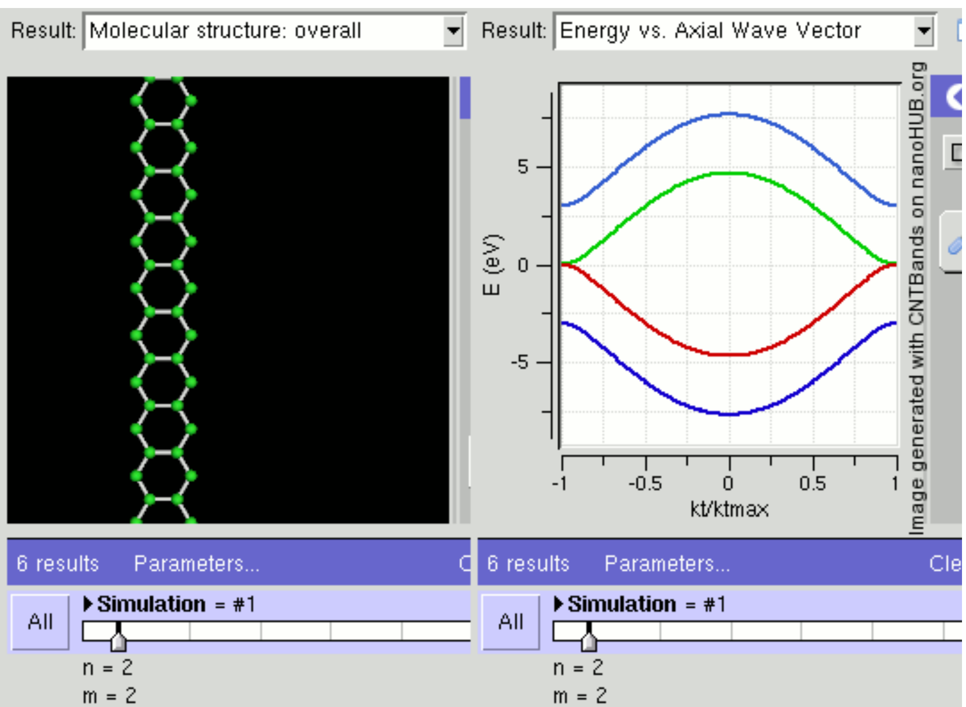
http://www.advancedsubstratenews.com/2010/12/
Advanced Substrate Corners
Posted by Professor Ru HUANG on December 8, 2010

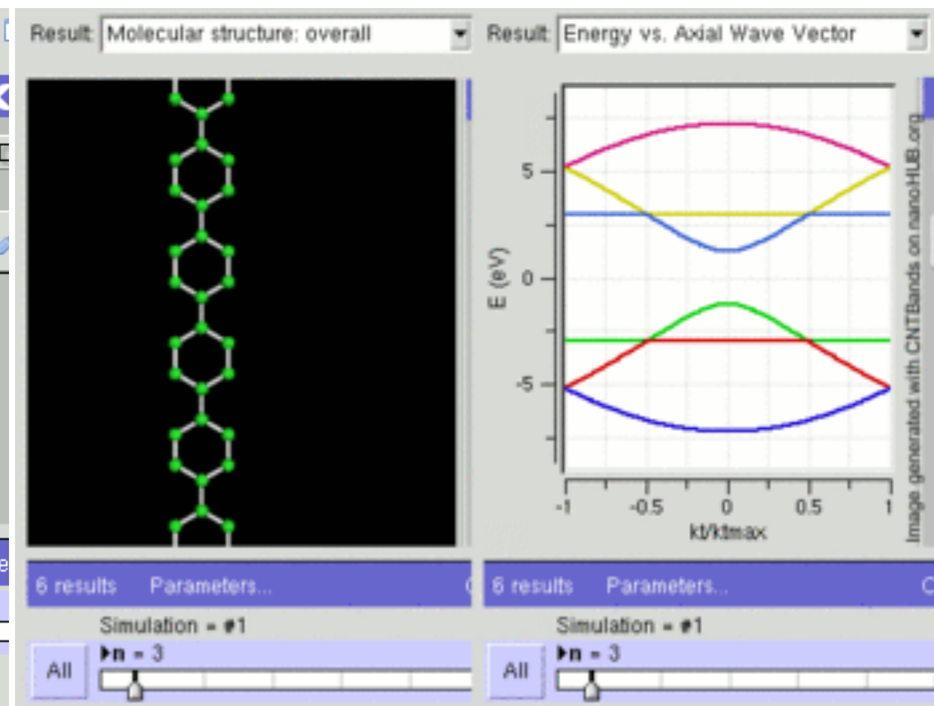
Carbon nanotube and Graphene Electronics



Gate-All-Around CNT



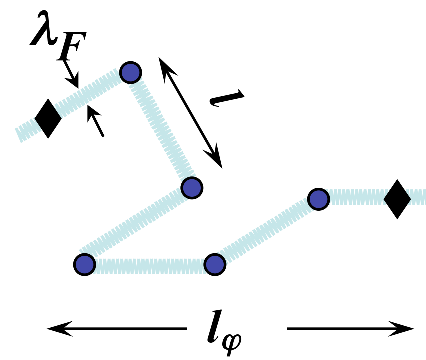




http://en.wikipedia.org/wiki/Graphene#Graphene_transistors

Do we see “quantum effect”
in the present-day semiconductor devices?

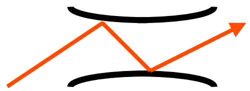
Characteristic length scales



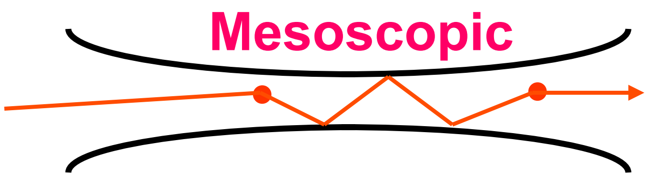
l : *elastic mean free path*

L_φ : *phase-breaking length*

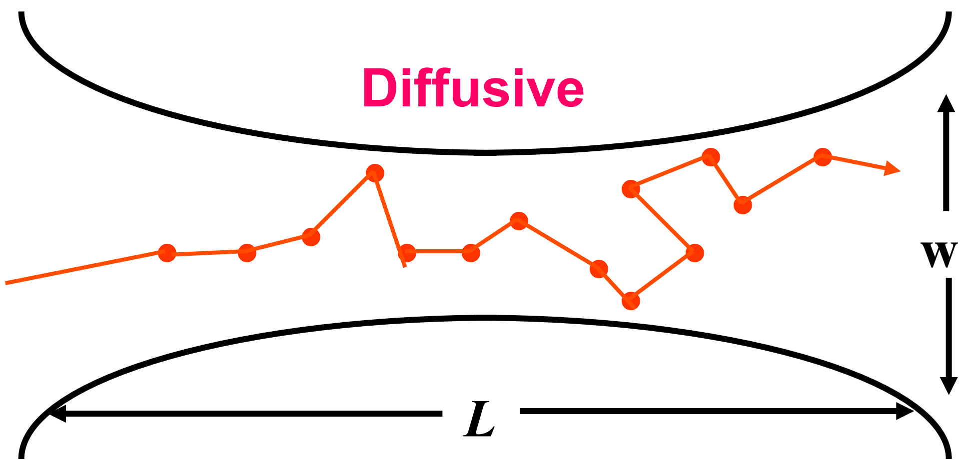
Ballistic



$(w, L) < l$

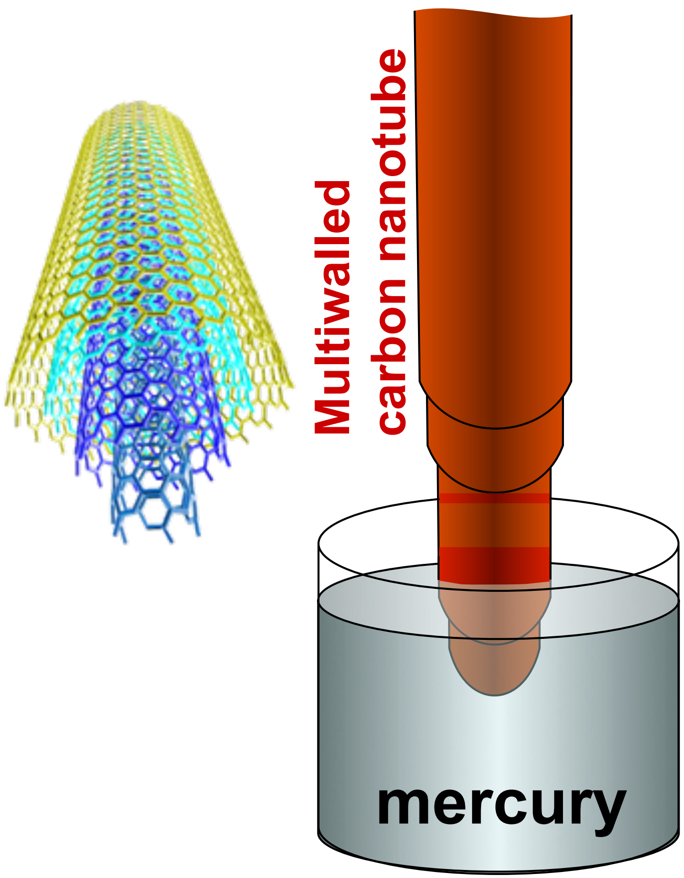


$l < (w, L) < L_\varphi$

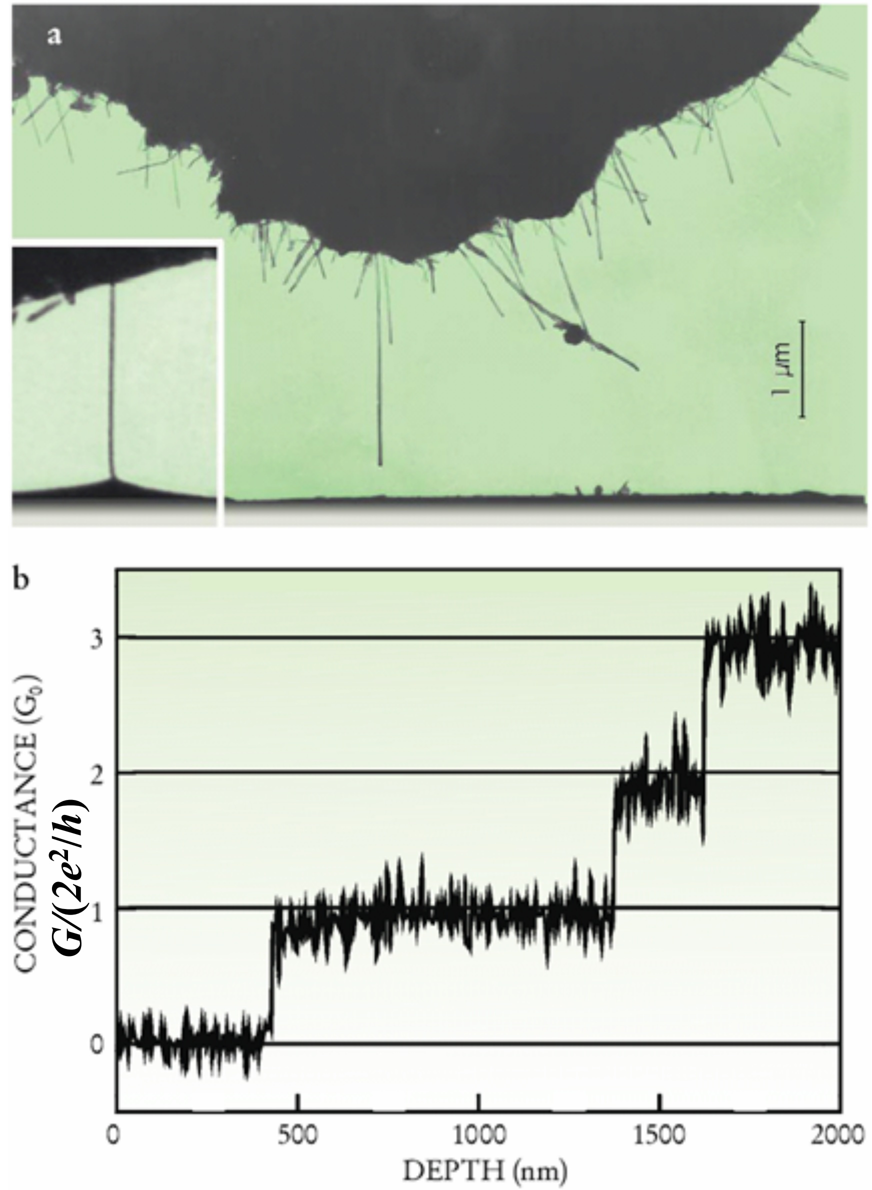


$L_\varphi << (w, L)$

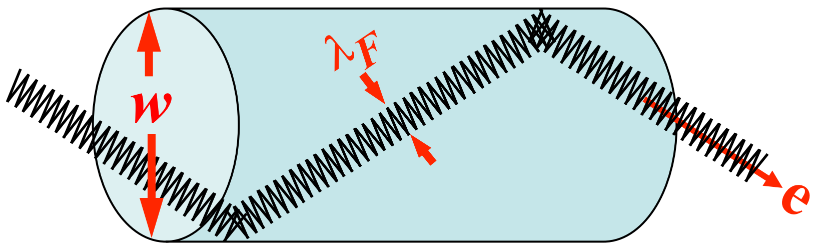
Ballistic transport:
Length independent conductance



Walt de Heer,
Georgia Institute of Technology



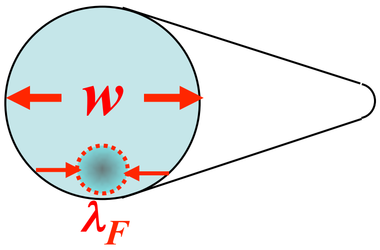
Ballistic system:



Devices with

1. Small diameter → only a few conduction channels

$$N \approx \pi w^2/\pi\lambda_F^2$$



2. weak *e-e* interaction
3. No impurity, no defect → no impurity scattering

$$G_0 = N\,2e^2/h$$

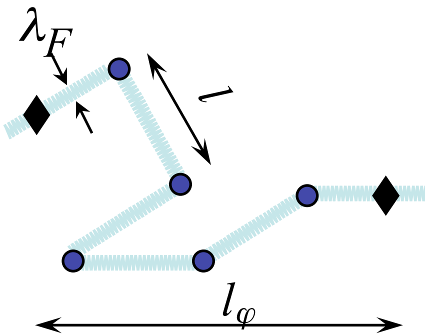
$$R_0 = \frac{1}{N}\frac{h}{2e^2} = \frac{R_Q}{N} \approx \frac{6.5k\Omega}{N}$$

R_Q: quantum resistance

Resistance independent of the length,
only determined by number of channels *N*

Resistance takes place at the macroscopic contacts

large



inelastic mean free path (phase breaking length) $l_\varphi = \sqrt{D\tau_\varphi}$

$l_\varphi \approx 0.1 \sim 1$ μm for metal

$l_\varphi \approx 0.2 \sim 20$ μm for GaAs

Ohm's law, drift eq., Einstein relation, Drude model

"usual" devices

Mesoscopic regime: to include quantum interference corrections

Weak localization, Aharonov-Bohm oscillation, Universal conductance fluctuations, persistent currents, …in small metallic wires, rings, Si MOSFETs, ..

elastic mean free path $l = v_F \tau$

$l \approx 0.01 \sim 0.1$ μm for metal

$l \approx 0.1 \sim 10$ μm for GaAs

Quantum ballistic transportation: the conductance fully determined by quantum mechanics

Conductance quantization, Quantum Hall effects in Quantum point contact, GaAs-based heterostructures

Fermi wavelength $\lambda_F = 2\pi/k_F$

$\lambda_F \approx 0.4$ nm for metal

$\lambda_F \approx 40$ nm for GaAs

Quantum size effects: Quantum confinement

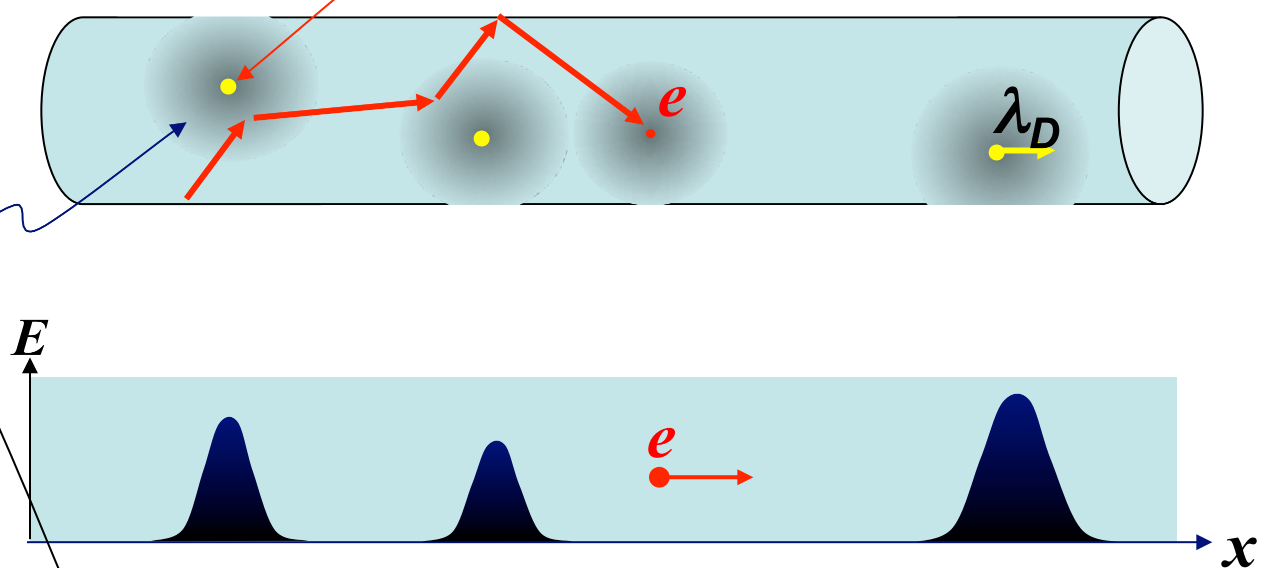
Energy level splitting in small metallic particles, GaAs-base quantum dots

small

Origin of resistance

electron transport in disorder systems

impurities / defects / dislocation /other carriers



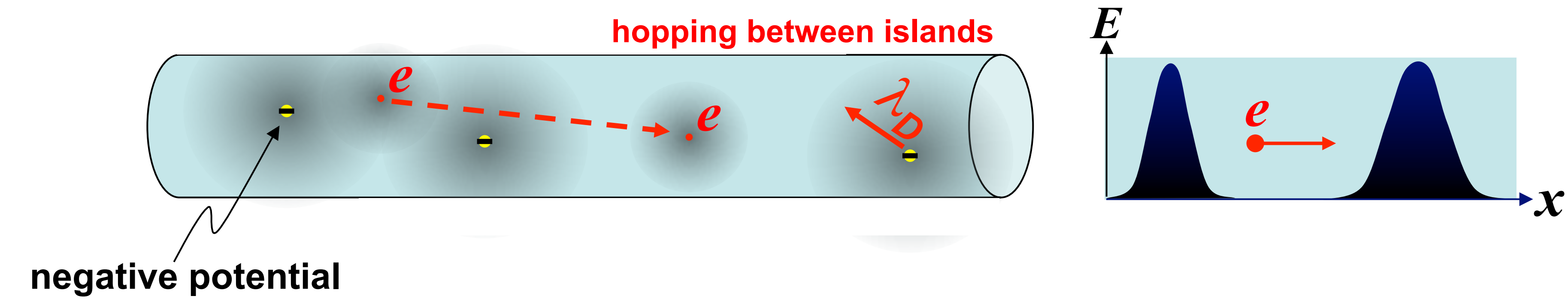
impurity potential

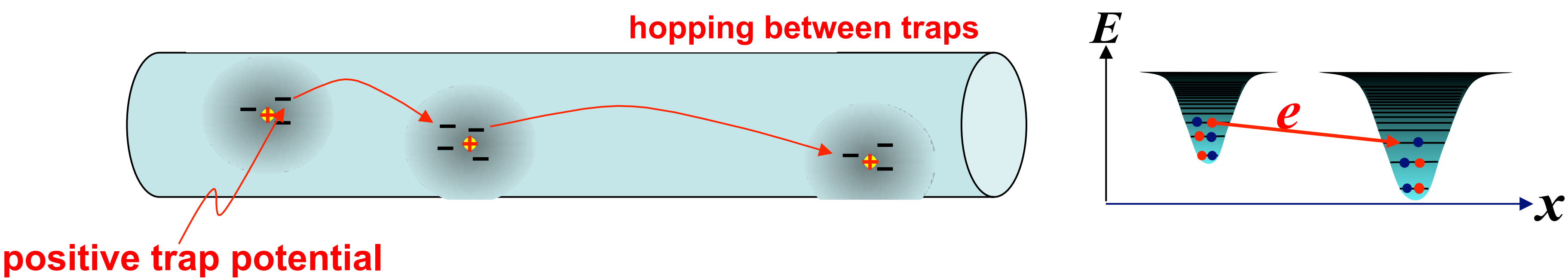
$$i\hbar \frac{\partial \Psi}{\partial t} = \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V \right) \Psi$$

Debye length $\lambda_D = \sqrt{\varepsilon k_B T / e^2 N_i}$

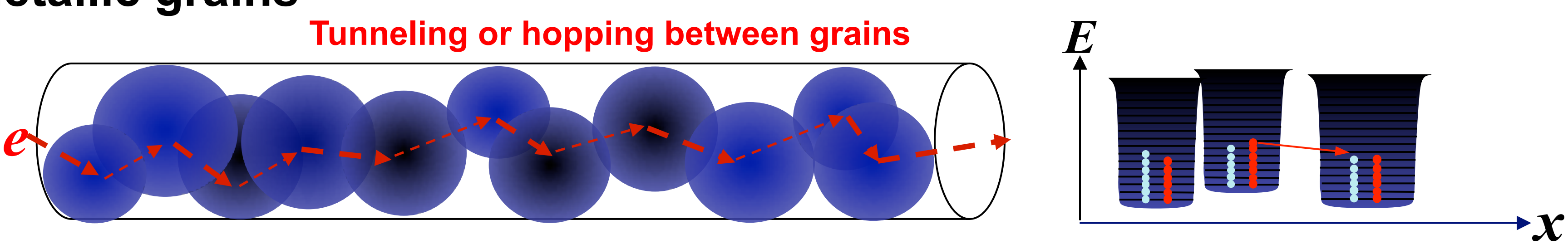
ε = dielectric permittivity,
k_B = Bolzmann constant,
T = absolute temperature,
e = electron charge

Hopping conduction in disordered systems





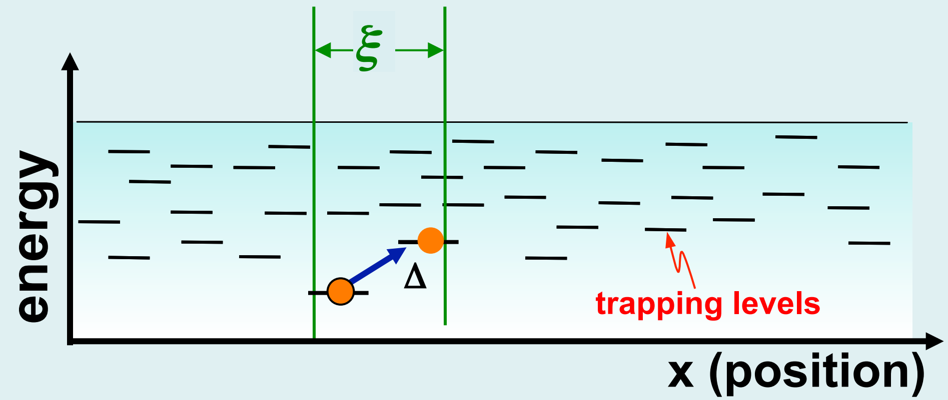
Metallic grains



Hopping transport

R_o increases with decreasing temperature

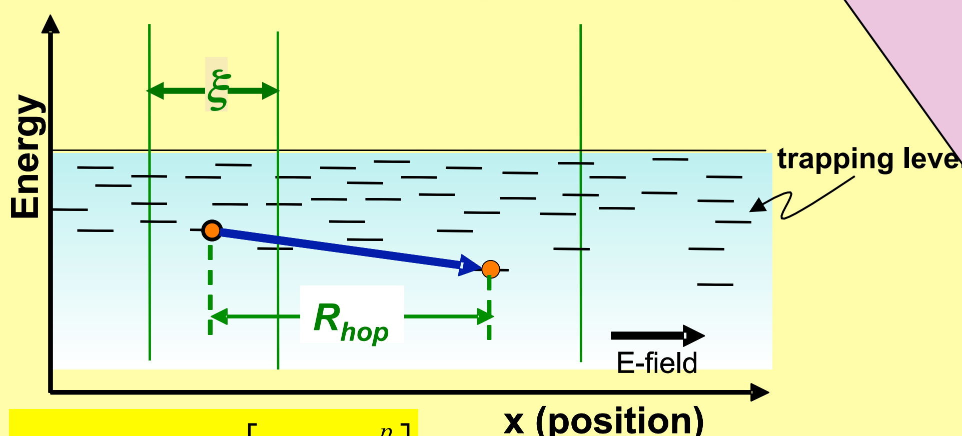
Nearest neighbor hopping (Arrhenius form)



$$R_0 = R_A \exp\left(\frac{\Delta}{k_B T}\right)$$

Presence of a **hopping barrier**; thermally activated hopping

Mott Variable range hopping



$$R_0 = R_A T^S \exp\left[\left(\frac{T_{Mott}}{T}\right)^p\right]$$

T_{Mott} = Mott characteristic temperature

$$p = \frac{1}{1+d}$$ d = system dimensionality

Efors-Shklovskii Variable range hopping

when Coulomb interaction is considered

$$R_0 = R_A T^S \exp\left[\left(\frac{T_{ES}}{T}\right)^{1/2}\right]$$

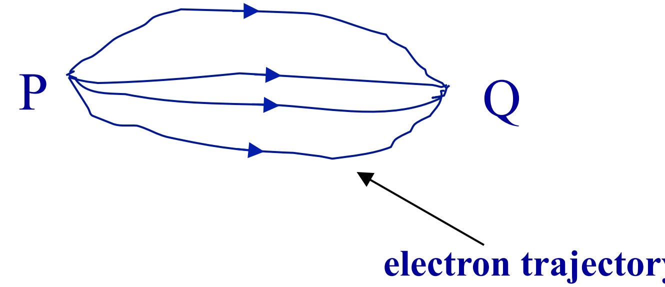
power-law dependence in DOS near E_F

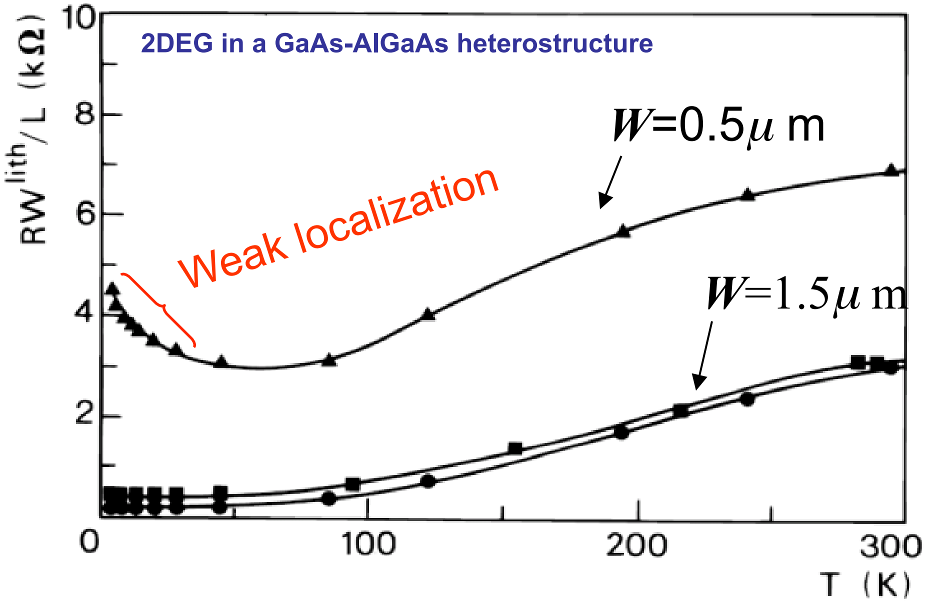
$$N(E) = N_0 |E - E_F|^{\gamma}$$

For 3D, γ=2

Nonlinear IV_b characteristics

Weak localization takes place for length scale < L_ϕ





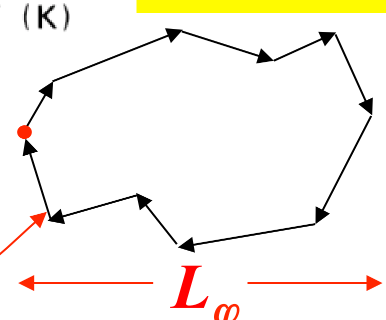
Probability for P → Q

$$W_{P\to Q} = \left|\sum_i A_i\right|^2 = \underbrace{\sum_i |A_i|^2}_{\text{Clasical diffusion}} + \underbrace{\sum_{i\neq j} A_i A_j^*}_{\text{Quantum interference}}$$

Clasical diffusion

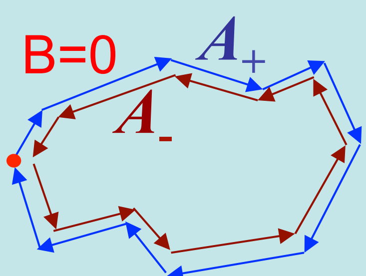
Quantum interference

"backscattering" event : (P=Q)



time reversal symmetry

elastic scattering



In zero field, B=0, A_+=A_-=A

Probability for back-scattering

$$W_{P\leftrightarrow P} = |A_+ + A_-|^2 = 4|A|^2$$

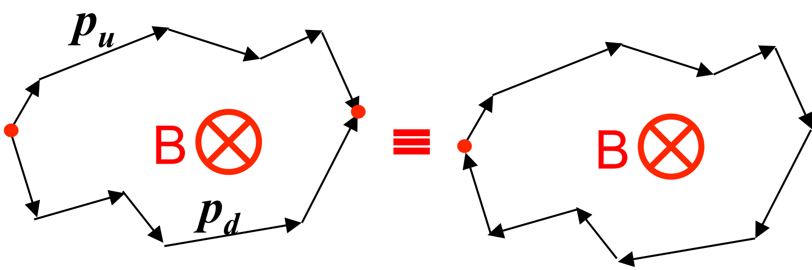
Suppression of weak localization by a magnetic field

In a field **B=∇×A**

Canonical momentum
$$p = mv - eA$$

electrons pick a phase φ
when traveling along a path P

$$\varphi = \frac{e}{\hbar}\int_P A\cdot dx$$



Magnetic field: break the time reversal symmetry

phase difference between two paths with the same ends

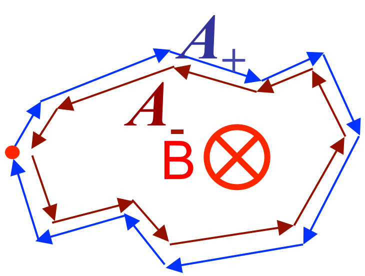
$$\delta\phi = \frac{1}{\hbar}\int_P^Q p_u\cdot dl_1 - \frac{1}{\hbar}\int_P^Q p_d\cdot dl_2 = \frac{e}{\hbar}\int_P^Q A_u\cdot dl_1 + \frac{e}{\hbar}\int_Q^P A_d\cdot dl_2 = \frac{e}{\hbar}\oint A\cdot dl$$

$$= \frac{e}{\hbar}\int(\nabla\times A)\cdot dS = \frac{e}{\hbar}\int BdS = 2\pi\frac{B\cdot S}{h/e}$$

= acquired phase around a loop

phase difference between A_+ and A_- =

$$2\delta\phi = 2\pi\frac{B\cdot S}{h/2e}$$

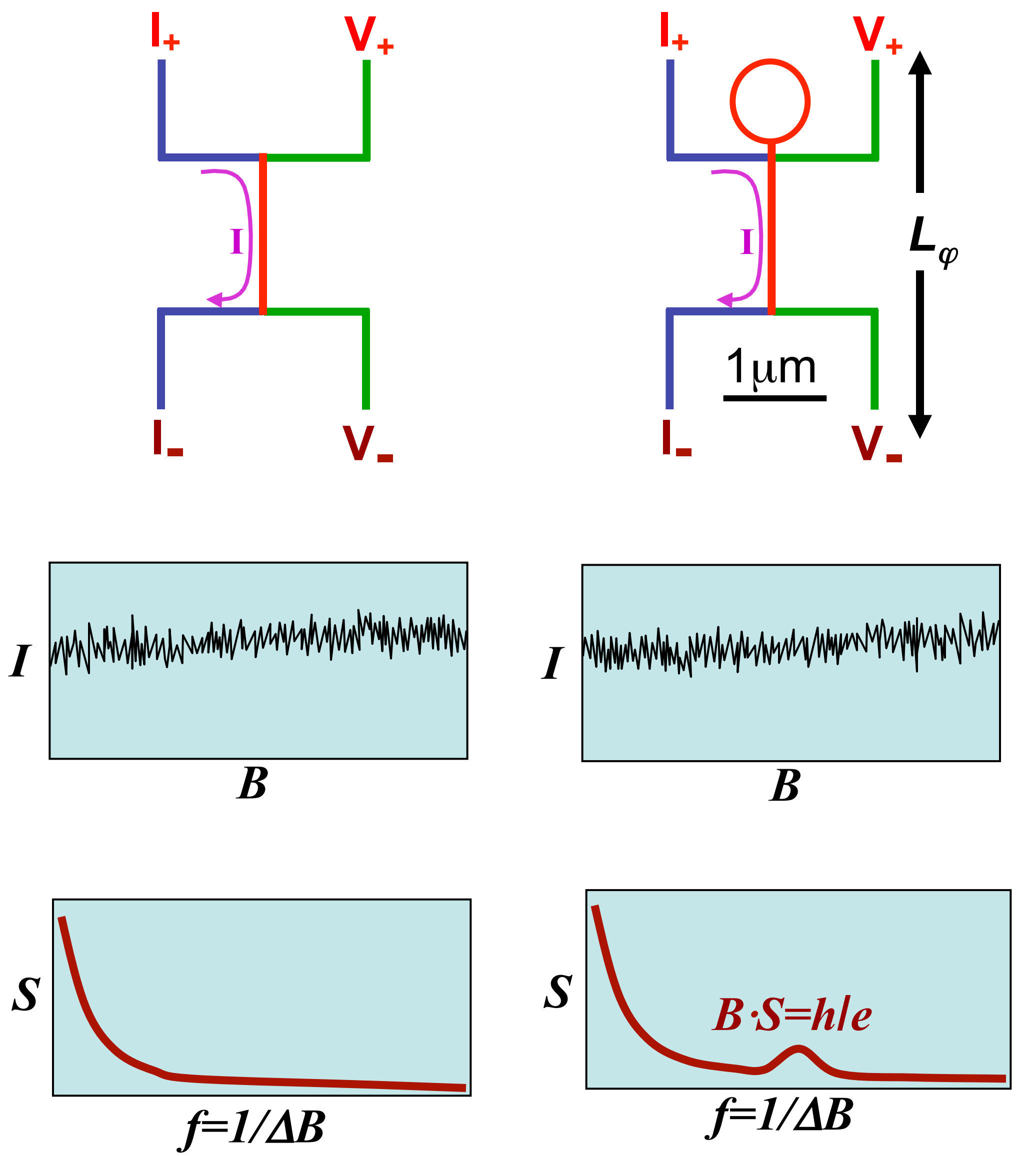


Probability for back-scattering

$$W_{P\leftrightarrow P} = 2|A|^2 + 2|A|^2\cos\left(2\pi\frac{B\cdot A}{h/2e}\right)$$

$$< 4|A|^2$$

Quantum correction to the conduction



The quantum correction to the conductance can be strongly influenced by phase coherent regions extending beyond the probes and outside the classical current paths

Observation of h/e Aharonov-Bohm Oscillations in Normal-Metal Rings

R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
(Received 27 March 1985)

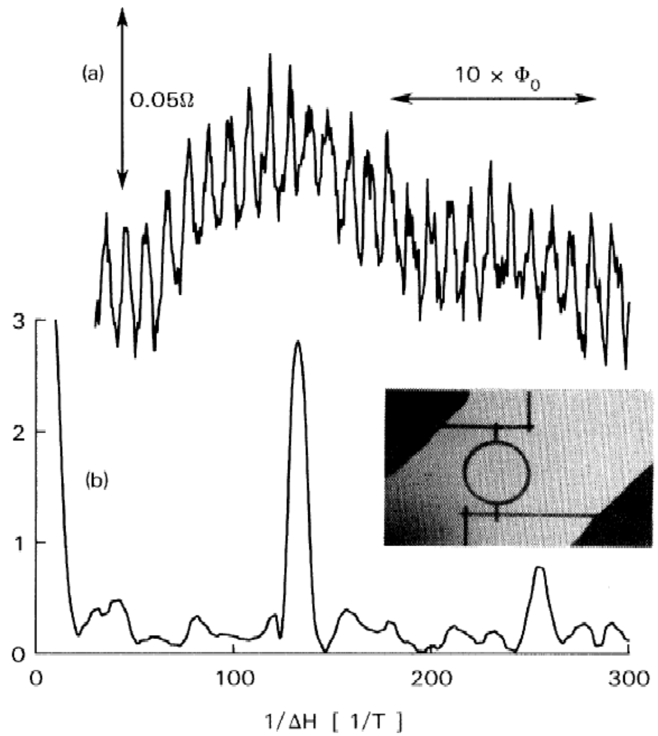


FIG. 1. (a) Magnetoresistance of the ring measured at $T=0.01$ K. (b) Fourier power spectrum in arbitrary units containing peaks at h/e and $h/2e$. The inset is a photograph of the larger ring. The inside diameter of the loop is 784 nm, and the width of the wires is 41 nm.

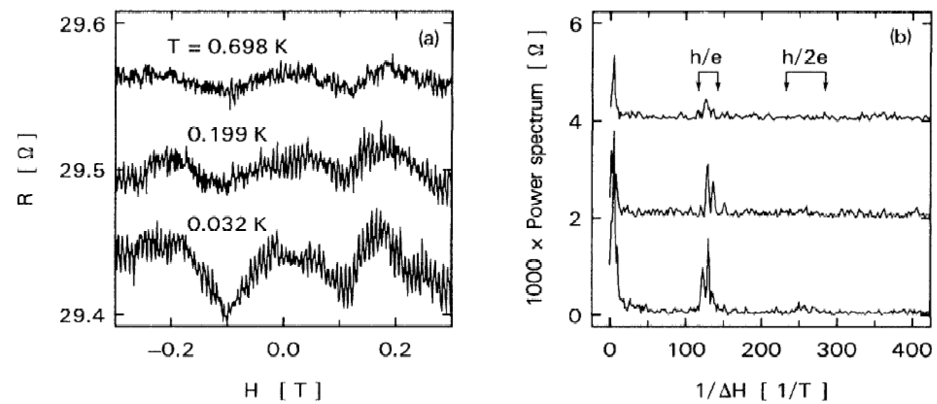
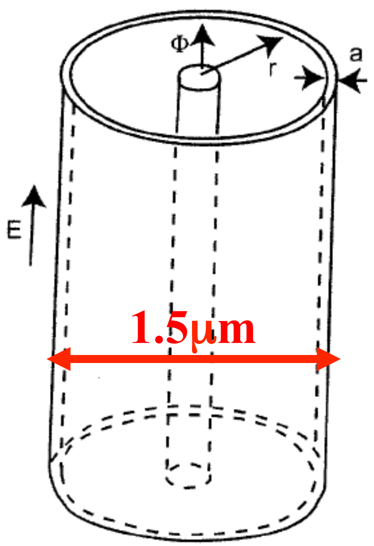


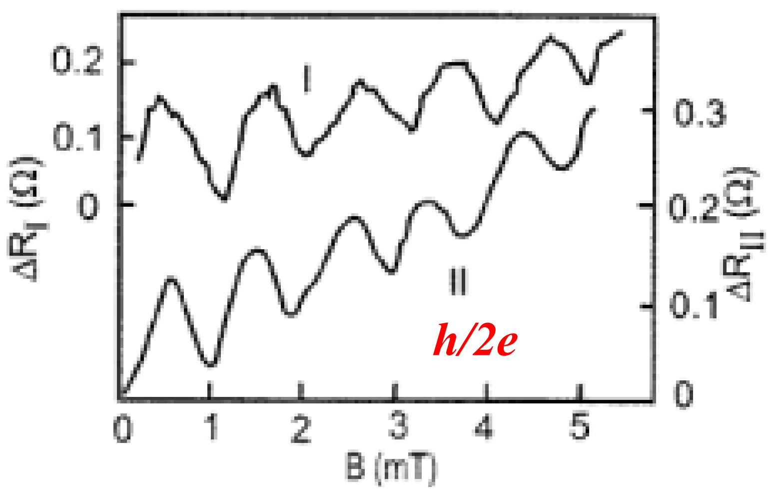
FIG. 2. (a) Magnetoresistance data from the ring in Fig. 1 at several temperatures. (b) The Fourier transform of the data in (a). The data at 0.199 and 0.698 K have been offset for clarity of display. The markers at the top of the figure indicate the bounds for the flux periods h/e and $h/2e$ based on the measured inside and outside diameters of the loop.

Experimental evidences :



ensemble of rings

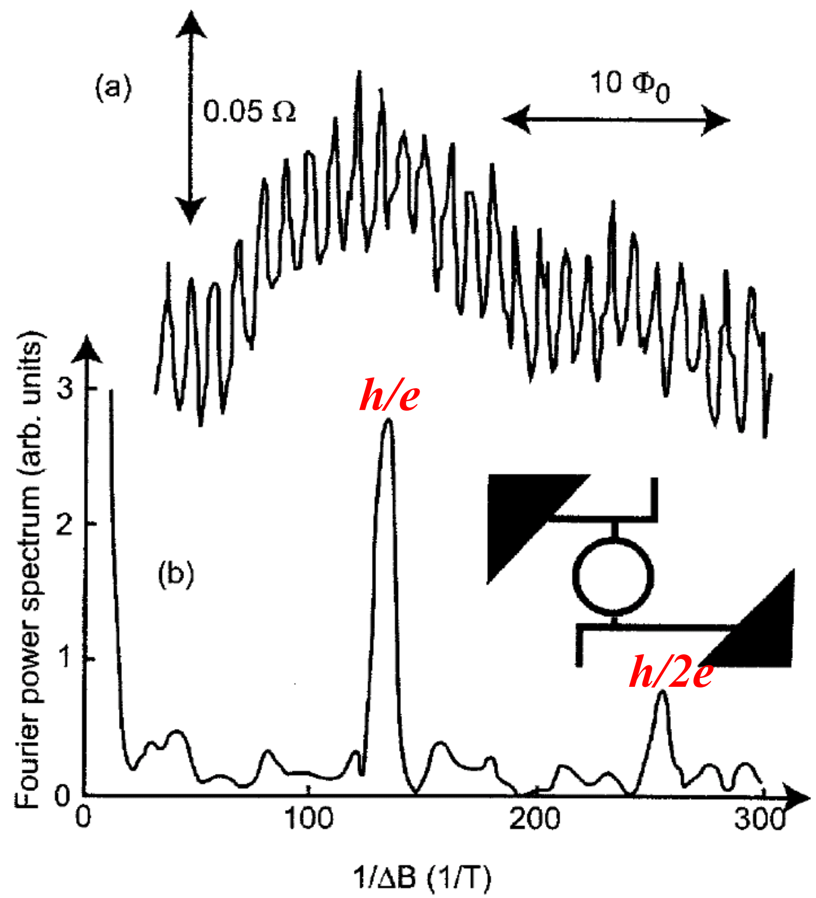
Insulating cylinder coated with a thin Mg film



Sharvin and Sharvin. JEPT Lett, 34, 272 (1981)

h/e **is averaged out**

Au-ring, inner diameter = 784nm, width = 41nm



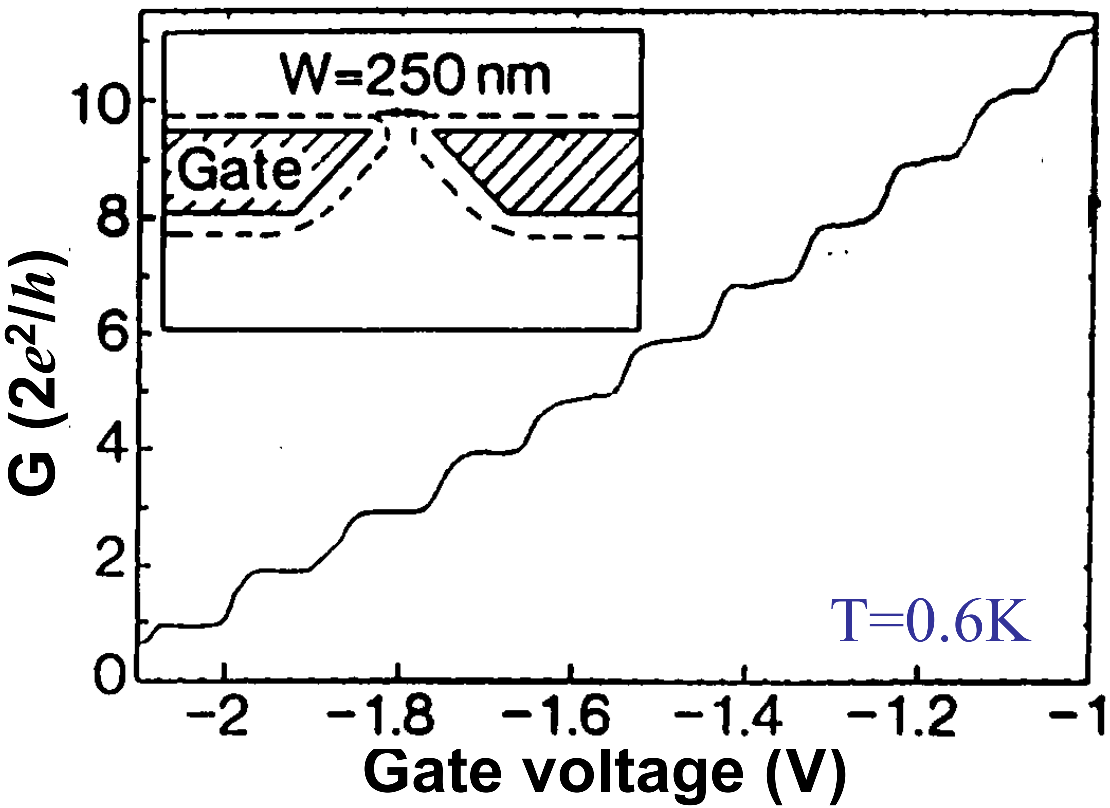
R.A. Webb et al. PRL, 54, 2696 (1985)

Conductance Quantization

Experiments on Quantum Point Contacts

B.J.van Wees et al. PRL 60, 848 (1988)

GaAs-AlGaAs heterostructure : 2DEG



$$G = \frac{2e^2}{h} \sum_n^N T_n(E_F)$$

$$T_n(E_F) = \sum_{n=1}^N |t_{m\to n}|^2$$

For adiabatic constriction

$$t_{m\to n} = \delta_{nm}$$

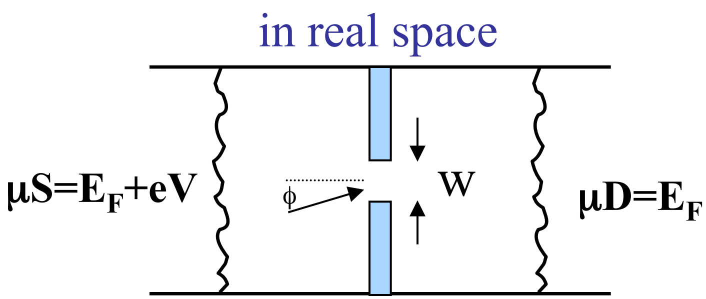
For abrupt constriction

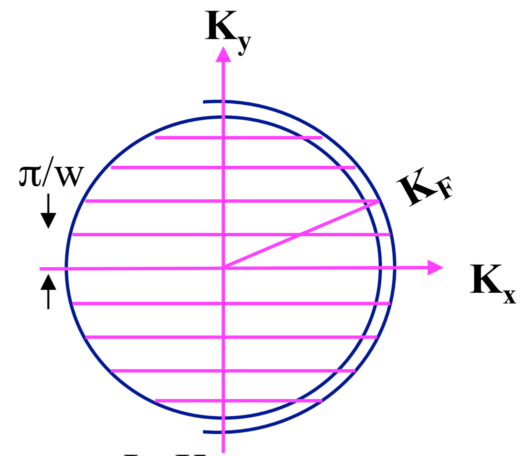
$$t_{m\to n} \neq 1$$

Optimized gate length

$$L_{opt} \approx 0.4\sqrt{\omega\lambda_F}$$

The origin of the conductance quantization





In K space

Diffusion current

transmission probability

$$I = e\sum_{n}^{N}\int_{E_F}^{E_F+eV} dE\,\rho_n(E)\,v_n(E)\,T_n(E) \qquad = \frac{2e}{h}eV\sum^{N} T_n(E_F)$$

1D Density of state

$$= \frac{1}{\pi}\left(\frac{dE_n}{dk}\right)^{-1}\frac{1}{\cos\phi}$$

Group velocity

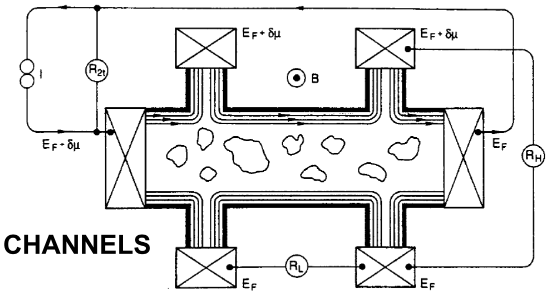
$$= \frac{1}{\hbar}\left(\frac{dE_n}{dk}\right)\cos\phi$$

$$N = \frac{w}{\lambda_F/2}$$

Landauer Formula for contact conductance

$$G = \frac{2e^2}{h}\sum_{n}^{N} T_n(E_F) = \frac{2e^2}{h}\frac{W}{\lambda_F/2}$$

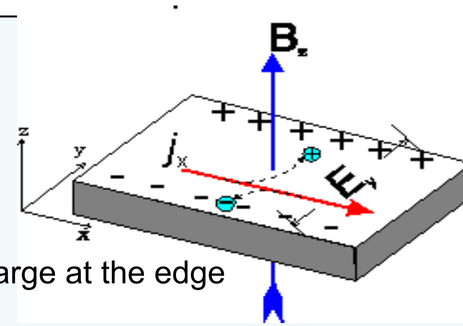
Quantum Hall Effect



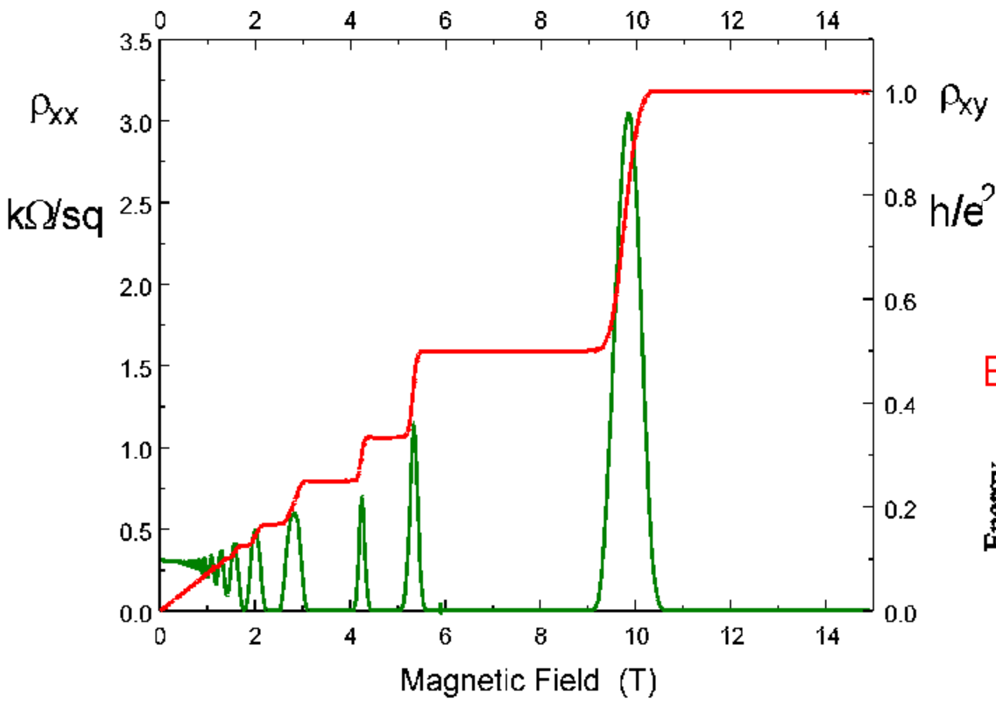
EDGE CHANNELS

Hall effect:

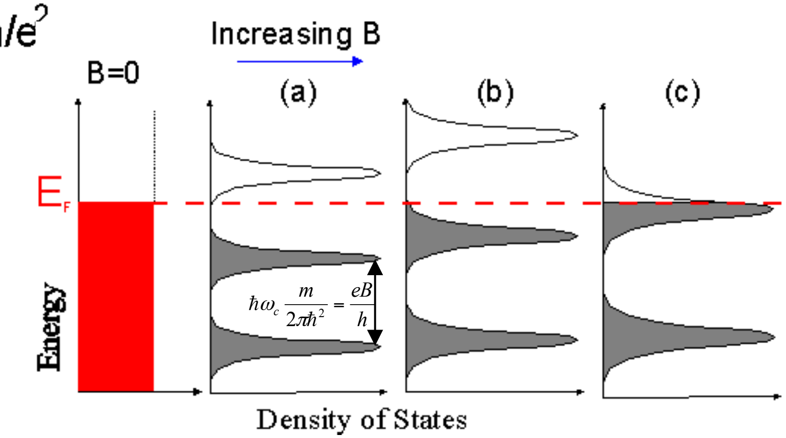
magnetic force =
electrostatic force
from the build up of charge at the edge



$R_H = E_y / j_x = 1/Nq$



$$\frac{mv^2}{r} = evB, v = r\omega_c \Rightarrow m\omega_c = eB$$



The value of resistance only depends on the fundamental constants of physics.

Resistance standard since 1990: h/ =25812.806 Ohms (precision)

http://www.warwick.ac.uk/~phsbm/qhe.htm

Energy Scales

Green light (550nm) = 2.25eV

Band gap (Si :1.1 *e*V, GaAs : 1.4 *e*V) (TiO_2 : 3.0 *e*V)

Superconductor energy gap (2Δ)
(Nb :3.0 me*V*, Pb : 2.7 me*V*, Al : 0.34 meV)

Coulomb interaction energy (charging energy)
($e^2/C \approx$ 0.5 meV ~ 10 me*V*) (6 K ~ 120 K)

Cyclotron energy for 2DEG at 1Tesla ($\hbar eB / m^*$ ~ 1.7 meV)

Level spacing
($\hbar^2 / m^* R^2$ ≈ 0.12 meV for a 100 nm 2DEG disc)
($2 / N_0(\varepsilon_F) V$ ≈ 0.15 meV for a 5 nm spherical metal particle)

Thermal energy ($k_B T$) 1 K ≈ 86 μeV

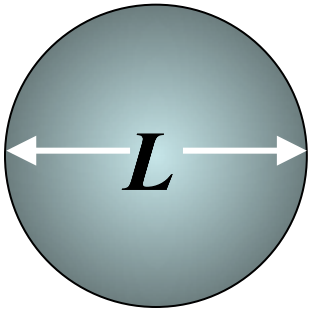
Spin Zeeman (gμB) 1T ~ 58 μeV

Tunneling coupling energy ($h\Gamma$) 10 μeV ($I \equiv e\Gamma$ = 0.4 nA)

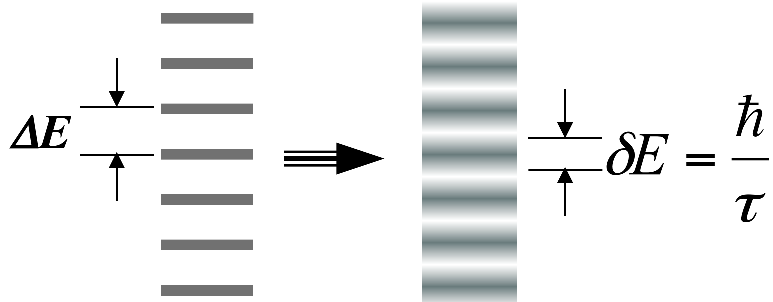
Josephson coupling energy $\left(\frac{\Delta}{2} \frac{R_Q}{R_T}\right)$ 10 μeV (for *Al junctions*, R_T = 64kΩ)

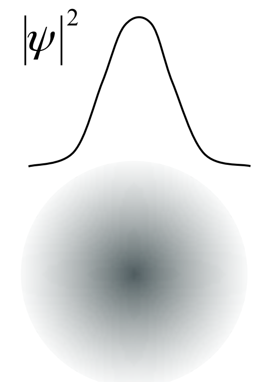
Microwave energy (*hf*) 1 GHz ≈ 4 μeV

Quantum origin of energy level broadening



broadening of quantized levels





τ = time for electron to travel through the grain

in ballistic regime: $\tau = L/v_F$

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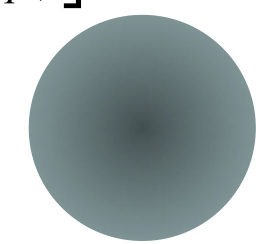
in diffusive regime: $\tau = L^2/D$

Einstein relation: $\sigma = e^2 D N_0(E_F) \Rightarrow \delta E \approx (\sigma h / e^2)\left[L^2 N_0(E_F)\right]^{-1}$

Level spacing : $\Delta E = [L^d N_0(E_F)]^{-1}$

Thouless ratio $g \equiv \dfrac{\delta E}{\Delta E} \approx \dfrac{h}{e^2}\sigma L^{d-2} = \dfrac{G}{e^2/h}$



$g < 1$ (R>26kΩ) ⇒ localized state

$g > 1$ (R<26kΩ) ⇒ extended state